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Rakhshanda Seyidzade
Baku State University
master student
raskhandaseyidzade@gmail.com

GENERAL FORM OF MULTISTEP METHODS AND SOME COMPARISONS OF THE VOLTERRA INTEGRAL EQUATION APPLIED TO THE SOLUTION OF THE INITIAL PROBLEM FOR FIRST-ORDER ODES

Abstract

Some methods are considered to be a connection between Volterra integral equations and differential equations. It is about some comparison of some methods for solving the initial problem for first-order differential equations, as well as the application of the forward running method in Voltaire's integral equation.Many methods of variable boundary theory are used in solving integral equations.These methods are considered to be a link between solving ordinary differential equations and Voltaire integral equations. Forward running method, hybrid method, etc. methods are special methods for solving these equations. There are many methods for solving Volterra-type equations.However, a numerical method that can ensure the reliability or regularity of the obtained results has not yet been. Therefore, it is important to establish some routine methods. Many transformations related to the calculation of the integral kernel are known. İt is possible to view some of them.

Keywords: *ODEs, Volterra type integral equation, hybrid method, forward running metod, quadrature method, first-order differential equations*

Rəxşəndə Seyidzadə
Bakı Dövlət Universiteti
magistrant
raskhandaseyidzade@gmail.com

Birinci dərəcəli əmsallar üçün ilkin məsələnin həllinə tətbiq edilən çoxmərhələli metodların ümumi görünüşü və Volterra inteqral tənliyinin bəzi müqayisələri

Xülasə

Bəzi üsullar Volterra inteqral tənlikləri ilə diferensial tənliklər arasında əlaqə hesab olunur. Söhbət birinci dərəcəli diferensial tənliklər üçün ilkin məsələnin həlli üçün bəzi üsulların bəzi müqayisəsindən, həmçinin Volterin inteqral tənliyində irəliyə doğru qaçış metodunun tətbiqindən gedir. İnteqral tənliklərin həllində dəyişən sərhəd nəzəriyyəsinin bir çox üsullarından istifadə olunur.

Bu üsullar adi diferensial tənliklərlə Volter inteqral tənliklərinin həlli arasında əlaqə hesab olunur. İrəli qaçış üsulu, hibrid metod və s. üsullar bu tənliklərin həlli üçün xüsusi üsullardır. Volterra tipli tənliklərin həlli üçün bir çox üsullar mövcuddur.

Lakin əldə edilən nəticələrin etibarlılığını və ya qanunauyğunluğunu təmin edə bilən ədədi üsul hələ də mövcud deyildir. Buna görə də bəzi rutin metodların yaradılması vacibdir. İnteqral nüvənin hesablanması ilə bağlı bir çox çevrilmələr məlumdur. Onlardan bəzilərinə baxmaq mümkündür.

Açar sözlər: *ODE-lər, Volterra tipli inteqral tənlik, hibrid metod, irəli qaçış metodu, kvadratura metodu, birinci dərəcəli diferensial tənliklər*

Introduction

Let's assume that Voltaire's integral equation of the second kind is given:

$$y(x) = f(x) + \int_{x_0}^{x} K\big(x, t, y(t)\big)dt \quad x_0 \le x \le X \qquad (*)$$

Since then, many scientists have tried to find approximate solutions of equation (*). We know that it is not easy to find the exact solution of equation (*). Recently, its numerical solutions have been much investigated Suppose that the equation (*) has a unrepeatable continuous solution on the segment $[x_0, X]$. To find the numerical solution of equation (*), we divide the segment into n equal parts. $x_m = x_0 + mh \;\; h > 0$. (Mehdiyeva, Imanova, 2019).

Quadrature method and its modifications are used in the numerical solution of many problems. Constant coefficients were established for the solution of equation (*). Sometimes such methods are used that it is possible to make a calculation by yourself. First the kernel of the integral is investigated. It is easy to see that if the integral's kernel, the function $K(x, y, z)$, is independent on x , the finding of the solution of equation (*) will be equivalent to finding of the Cauchy problem for first order ordinary differential equations (Mehdiyeva, Imanova, Ibrahimov, 2013: 4875-4890).

As you know, there are some classes of methods for solving the initial problem for first-order ODEs. Among them, the most popular numerical methods are Runge-Kutti methods, multi-step methods, forecast-correction methods, methods with anticipation, hybrid methods. Note that each of these methods is a separate object of research. Considering that in applications of multi-step methods, it is very often necessary to build stable methods with high accuracy. Some authors suggest using numerical methods with second derivatives. One of the simplest methods with second derivatives is the following method (Gear, 1965: 69-85).

$$y_{n+1} = y_n + \text{h}y_n^{ι} + h^2 y_n^{ιι}, \quad (1)$$

Which, when applied to the initial problem:

$$y^{ι} = f(x, y), \;\; y(x_0) = y_0, \;\; x_0 \le x \le x\,, \qquad (2)$$

Recorded

$$y_{m+1} = y_m + \text{h}\, f(x_m + y_m) + \frac{h^2}{2}\text{g}(x_m + y_m), \qquad (3)$$

It follows that when calculating the values of the function g (x, y), the number of computational work is doubled. But, the accuracy of the method increases by one, since method (1) has an accuracy of p>2, which follows from the expansions of the function in a Taylor series. Indeed, consider the following Taylor expansion:

$$\text{y (x + h)} = \text{y(x)} + h^{ι}\text{y}' \text{(x)} + \frac{h^2}{2} y^{ιι} \text{(x)} + \text{O}(\text{h}^3),$$

It is known that in this expansion the first two terms coincide with the known Euler method, so the local error of the Euler method is equal to $O(h^2)$. And the local error of method (1) is equal to $O(h^3)$ (Mehdiyeva, Ibrahimov, 2013: 314).

If we generalize method (1), then in one version we get:

$$\sum_{j=0}^{k} a_j y_{m+j} = h \sum_{j=0}^{k} b_j f_{m+j} + h^2 \sum_{j=0}^{k} c_j g_{m+j} \qquad (4)$$

Here coefficients $a_i b_i c_i$(0,1,2.......n) are some real numbers. Note that method (4) is usually studied in two forms. Methods with second derivatives in the general form are used in the form (4). However, depending on the tasks to be solved, this method can be represented as the following method:

$$\sum_{j>0}^{k} a_j y_{m+j} = h^2 \sum_{j>0}^{k} b_j g_{m+j} \quad . \qquad (5)$$

Note that the properties of methods (4) and (5) do not match. Therefore, they are a separate object of research.

The simplest method obtained from formula (5) can be written in the following form:

$$y_{m+1} = y_m + h^2 \mathrm{g}(x_m, y_m), \qquad (6)$$

As is known, one of the popular numerical methods for solving the Volterra equation is the quadrature method. However, like many other methods, the quadrature method also has some drawbacks. In their application to the solution of an integral equation of the Volterra type, the amount of computational work increases when moving from one point to the next. Here, a specially constructed method is proposed for solving integral equations freed from the above drawbacks (Mehdiyeva, Imanova, Ibrahimov, 2014: 352-356).

Consider the following non-linear Volterra equation:

$$y(x) = f(x) + \int_{x_0}^{x} K\big(x, s, y(s)\big) ds \quad (6)$$

It is clear that if we apply the quadrature method to the solution of the equation, we get:

$$y(x_m) = f(x_m) + h \sum_{j=0}^{m} b_j K(x_m, x_j, y_j) + P_n(x_m) \quad (7)$$

If we set up this method for calculations, we get:

$$y(x_{m+1}) = f(x_{m+1}) + h \sum_{j=o}^{m+1} b_j K(x_{m+1}, x_j, y_j) + P_n(x_{m+1}) \qquad (8)$$

Using comparisons of these methods, we observe that the number of calculations increases each time we move from one point to another. However, there are methods in which the number of uses does not increase in the calculation of the function when using it. For this purpose, consider the following method (Linz):

$$\sum_{m=0}^{k-l} a_i y_{n+m} = h \sum_{m=0}^{k} \sum_{j=m}^{k} b_m^j K(x_j, x_m, y_m) \ (9)$$

Note that if , method (9) is considered an obvious method , if , then method (9) is usually called non-obvious , if .

These methods use data from current and previous split points. Recently, however, methods have been used that use information about the solution at later points. When there are these methods, it is called forward running method derived from method (9). However, this does not mean that forward running methods are a special case of implicit or explicit methods. Note that forward running methods are an object of free research In this method, it is easy to determine that the values of the solution of the problem studied at the previous, current and subsequent points are used.

Thus, we observe that the point where we are looking for the value of the solution of the studied problem depends on the value of the function at the previous and subsequent points, that is, these values are located symmetrically. This method was applied to the solution of the model integral equation for example. The obtained result is consistent with the theoretical ones (Butcher, 1965: 124-135).

Conclusion

The aim of this work is to apply the method of constructing solutions to equation (*), the study of the solution of integral equations of Volterra type with symmetrical borders,to get more accurate results it is necessary to use secret methods, but their application to the solution of specific equations have difficulty choosing appropriate methods of forecasting.as a result, application to solving of integral equations with symmetric variable boundaries method for constructing solutions to the equation does not take place mechanically. According to this we consider the use of multistep methods with constant coefficients to solving integral equations with symmetric boundaries of Volterra type (Yanenko, 1967).

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